Input Impedance of a Probe-Excited Semi-Infinite Rectangular Waveguide with Arbitrary Multilayered Loads: Part I—Dyadic Green's Functions

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***Abstract*—Part I of this paper presents both the electric and the magnetic types of dyadic Green's functions defined for electromagnetic fields due to electric and magnetic current sources in a semi-infinite rectangular waveguide filled with arbitrary multilayered media. Applying the principle of scattering superposition, the dyadic Green's functions in each of the multiple loads are constructed in general for such EM current sources located in an arbitrary layer of the waveguide. Analytical expressions of the scattering dyadic Green's functions' coefficients are obtained in terms of transmission matrices. To demonstrate how the method presented is used and how the results are obtained for some special cases, a semi-infinite rectangular waveguide with one load is considered. The dyadic Green's functions and their coefficients in such a case are derived in closed form by reducing the general formulae of the dyadic Green's functions for the arbitrary multiple case to those for the special case concerned. Further comparison of the dyadic Green's functions obtained here with previous publications shows good agreement, demonstrating the applicability of the results presented here. Part II of this paper will present a full-wave numerical analysis of a probe with both electric and magnetic current distributions.**

I. INTRODUCTION

THE DYADIC Green's function technique is an efficient method for solving boundary-value problems in electromagnetics. Such a technique has also been extensively applied to scattering and excitation problems associated with rectangular cavities and waveguides [1], [2]. The electric and magnetic types of dyadic Green's functions have been constructed for the rectangular cavity by Tai [1], Collin [2], Rahmat-Samii [3], Liang *et al.* [4], and Li *et al.* [5]; for semi-infinite rectangular waveguide by Jarem [6], [7], Tai [1], Collin [2], Balanis [8], and Li *et al.* [5]; and for infinite rectangular waveguide by Tai [1], Collin [2], Rahmat-Samii [3], Xu [9], and Li *et al.* [5].

Recently, a $\hat{y}\hat{y}$-component of the electric type of dyadic Green's functions was given by Jarem [7] for the analysis of a probe-sleeve fed rectangular waveguide with one load. In Part I of this paper, the dyadic Green's functions in a semi-infinite rectangular waveguide with arbitrary multiple loads are presented. Since the magnetic type of dyadic Green's function cannot be converted directly from the electric type by simple and conventional substitutions [10], both the electric and the magnetic types of dyadic Green's functions are derived in this paper. The coefficients of the scattering dyadic Green's functions for N-multiple loads are calculated analytically and given in the recurrence form of a transmission matrix. A semi-infinite rectangular waveguide with one load is used to demonstrate how the method presented here is applied to obtain the results for special cases by simple reduction. Good agreement between the new results reduced here from the general case to the special case and some results previously published elsewhere has been found.

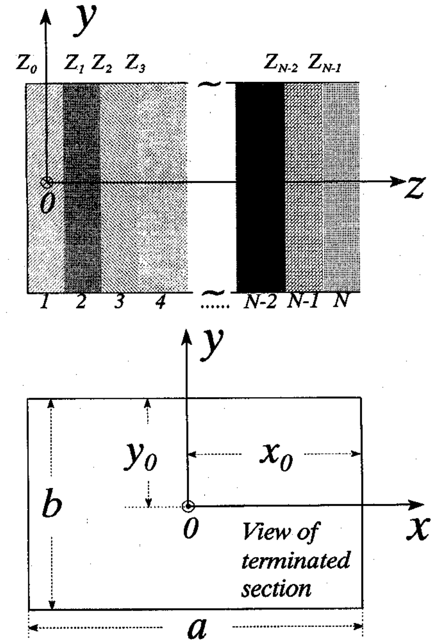

Fig. 1. Geometry of the semi-infinite rectangular waveguide filled with multilayered media of different dielectric constants.

II. FUNDAMENTAL PROBLEM

The geometry of the semi-infinite rectangular waveguide filled with arbitrary multi-layered media of different dielectric constants is shown in Fig. 1, where the waveguide is divided

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along its length into N regions numbered as $1, 2, \cdots, N$, respectively. The electromagnetic radiation fields, $\boldsymbol{E}_f$ and $\boldsymbol{H}_f$ $(f = 1, 2, \cdots, N)$, in the fth region of the semi-infinite rectangular waveguide, which are contributed by the electric and magnetic current distributions $\boldsymbol{J}_s$ and $\boldsymbol{M}_s$ $(s = 1, 2, \cdots, N)$ located in the sth region, are governed by

$$\nabla \times \nabla \times \boldsymbol{E}_f - k_f^2 \boldsymbol{E}_f = (i\omega\mu_f \boldsymbol{J}_s - \nabla \times \boldsymbol{M}_s)\delta_f^s, \quad (1a)$$
$$\nabla \times \nabla \times \boldsymbol{H}_f - k_f^2 \boldsymbol{H}_f = (i\omega\varepsilon_f \boldsymbol{M}_s + \nabla \times \boldsymbol{J}_s)\delta_f^s \quad (1b)$$

where without any loss of generality, the propagation constant in the fth region of the waveguide is designated as $k_f = \omega\sqrt{\mu_f \varepsilon_f (1 + i\sigma_f/\omega\varepsilon_f)}$, and ε_f, μ_f, and σ_f are the permittivity, permeability, and conductivity in the fth region, respectively. A time-dependence $\exp(-i\omega t)$ is assumed throughout the paper for the construction of dyadic Green's functions and field calculations. It is noted that symbols in both bold and italic face are used to denote vectors while those in bold face only are reserved for matrices.

To obtain the electromagnetic fields due to the electric and magnetic current sources, we first construct the electric and magnetic types of dyadic Green's functions $\overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r}, \boldsymbol{r}')$ and $\overline{\boldsymbol{G}}_m^{(fs)}(\boldsymbol{r}, \boldsymbol{r}')$ [5], [10], [11], respectively. These two dyadics satisfy the following equations:

$$\nabla \times \nabla \times \overline{\boldsymbol{G}}_{\substack{e\\m}}^{(fs)}(\boldsymbol{r}, \boldsymbol{r}') - k_f^2 \overline{\boldsymbol{G}}_{\substack{e\\m}}^{(fs)}(\boldsymbol{r}, \boldsymbol{r}') = \overline{\boldsymbol{I}}\delta(\boldsymbol{r} - \boldsymbol{r}'), \quad (2)$$

where $\delta(\boldsymbol{r} - \boldsymbol{r}')$ represents a Dirac delta function, and $\overline{\boldsymbol{I}}$ denotes the unit dyad operator. The boundary conditions satisfied by the electric and magnetic types of Green dyadics are given by the following formulae on the walls:

$$\hat{\boldsymbol{n}} \times \overline{\boldsymbol{G}}_e^{(1s)}(\boldsymbol{r}, \boldsymbol{r}') = 0, \quad \hat{\boldsymbol{n}} \times \nabla \times \overline{\boldsymbol{G}}_m^{(1s)}(\boldsymbol{r}, \boldsymbol{r}') = 0 \quad (3a)$$

and at the interfaces $z = z_\ell$ $(\ell = 1, 2, \cdots, N - 2)$

$$\hat{\boldsymbol{z}} \times \overline{\boldsymbol{G}}_{\substack{e\\m}}^{(\ell s)}(\boldsymbol{r}, \boldsymbol{r}') = \hat{\boldsymbol{z}} \times \overline{\boldsymbol{G}}_{\substack{e\\m}}^{[(\ell+1)s]}(\boldsymbol{r}, \boldsymbol{r}'), \quad (3b)$$
$$\frac{1}{\wp_\ell}\hat{\boldsymbol{z}} \times \nabla \times \overline{\boldsymbol{G}}_{\substack{e\\m}}^{(\ell s)}(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{\wp_{\ell+1}}\hat{\boldsymbol{z}} \times \nabla \times \overline{\boldsymbol{G}}_{\substack{e\\m}}^{[(\ell+1)s]}(\boldsymbol{r}, \boldsymbol{r}') \quad (3c)$$

where $\wp$ denotes μ for the *electric* type of dyadic Green's functions or the e-modes and ε for the *magnetic* type of dyadic Green's functions or the o-modes. Furthermore, the Sommerfeld radiation condition at $z \to \infty$ must be satisfied, i.e.

$$\lim_{z\to\infty} [\nabla_z \times \overline{\boldsymbol{G}}_{\substack{e\\m}}^{(Ns)}(\boldsymbol{r}, \boldsymbol{r}') - i\gamma\hat{\boldsymbol{z}} \times \overline{\boldsymbol{G}}_{\substack{e\\m}}^{(Ns)}(\boldsymbol{r}, \boldsymbol{r}')] = 0 \quad (3d)$$

where $\hat{\boldsymbol{z}}$ indicates the propagation direction of the electromagnetic waves in the waveguide, γ is the z-directional propagation constant, and the differential operator ∇_z is given by $\hat{\boldsymbol{z}}\partial/\partial z$ for an infinite waveguide.

After obtaining the electromagnetic types of dyadic Green's functions, the electromagnetic fields $\boldsymbol{E}_f$ and $\boldsymbol{H}_f$ in the fth region due to the electric and magnetic current sources $\boldsymbol{J}_s$ and $\boldsymbol{M}_s$ in the sth region can be obtained in terms of the electric and magnetic types of Green dyadic integrals as follows:

$$\boldsymbol{E}_f(\boldsymbol{r}) = i\omega\mu_s \iiint_V \overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{J}_s(\boldsymbol{r}')\, dV' - \iiint_V \nabla \times [\overline{\boldsymbol{G}}_m^{(fs)}(\boldsymbol{r}, \boldsymbol{r}')] \cdot \boldsymbol{M}_s(\boldsymbol{r}')\, dV', \quad (4a)$$
$$\boldsymbol{H}_f(\boldsymbol{r}) = \iiint_V \nabla \times [\overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r}, \boldsymbol{r}')] \cdot \boldsymbol{J}_s(\boldsymbol{r}')\, dV' + i\omega\varepsilon_s \iiint_V \overline{\boldsymbol{G}}_m^{(fs)}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{M}_s(\boldsymbol{r}')\, dV' \quad (4b)$$

where V identifies the volume occupied by the sources in the sth region and the subscripts, e and m, denote the *e*lectric and *m*agnetic types of Green dyadics.

III. Electromagnetic Types of Dyadic Green's Functions

To obtain the electromagnetic types of dyadic Green's functions in the spectral domain, two methods are available, namely, 1) the direct dyad representation in terms of the unit coordinate vectors and 2) the indirect dyad representation in terms of vector wave eigenfunction expansions. In our present work, the latter will be applied to construct the electromagnetic types of dyadic Green's functions.

To do so, the scalar eigenfunction $\psi_{\substack{e\\o}mn}(\gamma)$ $(m, n = 0, 1, 2, \cdots)$ is derived by the method of separation of variables. In the case of the rectangular waveguide, the function has the following form:

$$\psi_{\substack{e\\o}mn}(\gamma) = \begin{Bmatrix} \cos\dfrac{n\pi(x + x_0)}{a} \cos\dfrac{m\pi(y + y_0)}{b} \\ \sin\dfrac{n\pi(x + x_0)}{a} \sin\dfrac{m\pi(y + y_0)}{b} \end{Bmatrix} e^{i\gamma z}. \quad (5)$$

Furthermore, the rectangular vector wave eigenfunction can be expressed in terms of the scalar eigenfunction (5) as follows:

$$\boldsymbol{M}_{\substack{e\\o}mn}(\gamma) = \nabla \times [\psi_{\substack{e\\o}mn}(\gamma)\hat{\boldsymbol{z}}], \quad (6a)$$
$$\boldsymbol{N}_{\substack{e\\o}mn}(\gamma) = \frac{1}{k}\nabla \times \nabla \times [\psi_{\substack{e\\o}mn}(\gamma)\hat{\boldsymbol{z}}] \quad (6b)$$

where k_c^2 is given by the following definition:

$$k^2 = \gamma^2 + \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 = \gamma^2 + k_c^2. \quad (7)$$

The orthogonality of these vector wave functions is maintained not only among themselves but also with respect to each other as they are integrated over all the values of x, y and z as shown in [1]. With the help of these vector wave functions, the dyadic Green's functions can be constructed with the outgoing and incoming wave vectors.

The magnetic type of dyadic Green's functions due to the magnetic current source can usually be converted directly from the electric type due to the electric current source by the simple substitutions $\boldsymbol{E} \to \boldsymbol{H}, \boldsymbol{H} \to -\boldsymbol{E}, \boldsymbol{J} \to \boldsymbol{M}, \boldsymbol{M} \to -\boldsymbol{J}, \mu \to \varepsilon$, and $\varepsilon \to \mu$ [10]. However, it was found in [5] that for the rectangular cavities and waveguides, an additional substitutions, i.e., even mode $(e) \to$ odd mode (o) and odd mode $(o) \to$ even mode (e), should be made in the derivations. This paper presents both the electric and magnetic types of dyadic Green's functions for the rectangular cavities and waveguides. To present both electric and magnetic types in the ${}^{e}_{m}$ format requires no extra space, but gives readers a straightforward expression to work with.

In deriving the dyadic Green's functions at the source regions, as mentioned by Rahmat-Samii [3], care must be exercised. At the source regions, singularities exist and therefore need to be taken into account in the representation of dyadic Green's functions. In fact, a delta function can be used to express the singularity term. In addition, the principle of scattering superposition may be applied in the construction. This principle states that the dyadic Green's function $\overline{\boldsymbol{G}}_{\substack{e\\m}}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ can be considered as the sum of the unbounded (with respect to z-direction) dyad $\overline{\boldsymbol{G}}_{\substack{e\\m}0}(\boldsymbol{r},\boldsymbol{r}')$ and the scattering Green dyadic $\overline{\boldsymbol{G}}_{\substack{e\\m}S}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ contributed by the interfaces perpendicular to the z-direction, i.e.

$$\overline{\boldsymbol{G}}_{\substack{e\\m}}^{(fs)}(\boldsymbol{r},\boldsymbol{r}') = \overline{\boldsymbol{G}}_{\substack{e\\m}0}(\boldsymbol{r},\boldsymbol{r}')\delta_f^s + \overline{\boldsymbol{G}}_{\substack{e\\m}S}^{(fs)}(\boldsymbol{r},\boldsymbol{r}') \tag{8}$$

where δ_f^s denotes Kronecker delta, and the unbounded electromagnetic types of dyadic Green's functions $\overline{\boldsymbol{G}}_{\substack{e\\m}0}(\boldsymbol{r},\boldsymbol{r}')$ consisting of the singularity and the principal value is, according to the Sommerfeld radiation conditions in (3d), given by

$$\begin{aligned}
&\overline{\boldsymbol{G}}_{\substack{e\\m}0}(\boldsymbol{r},\boldsymbol{r}')\\
&\quad = -\frac{\hat{z}\hat{z}\delta(\boldsymbol{r}-\boldsymbol{r}')}{k_s^2} + PV_\delta \overline{\boldsymbol{G}}_{\substack{e\\m}0}(\boldsymbol{r},\boldsymbol{r}')\\
&\quad = -\frac{\hat{z}\hat{z}\delta(\boldsymbol{r}-\boldsymbol{r}')}{k_s^2} + \frac{i}{ab}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{2-\delta_0}{\gamma_s k_c^2}\\
&\quad\cdot\begin{cases}\boldsymbol{M}_{\substack{e\\o}mn}(\gamma_s)\boldsymbol{M}'_{\substack{e\\o}mn}(-\gamma_s) + \boldsymbol{N}_{\substack{o\\e}mn}(\gamma_s)\boldsymbol{N}'_{\substack{o\\e}mn}(-\gamma_s)\\ \qquad z \geq z',\\ \boldsymbol{M}_{\substack{e\\o}mn}(-\gamma_s)\boldsymbol{M}'_{\substack{e\\o}mn}(\gamma_s) + \boldsymbol{N}_{\substack{o\\e}mn}(-\gamma_s)\boldsymbol{N}'_{\substack{o\\e}mn}(\gamma_s)\\ \qquad z < z',\end{cases}\\
&\qquad (-\infty < z < \infty; -\infty < z' < \infty)
\end{aligned} \tag{9}$$

in which δ_0 ($= 1$ for m or $n = 0$, and 0 otherwise) denotes the Kronecker delta and $\gamma_s^2 = k_s^2 - k_c^2$ and PV_δ represents the principal value. The scattering Green dyadic $\overline{\boldsymbol{G}}_{\substack{e\\m}s}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ can be presented as follows [5], [10]:

$$\begin{aligned}
\overline{\boldsymbol{G}}_{\substack{e\\m}S}^{(fs)}(\boldsymbol{r},\boldsymbol{r}') &= \frac{i}{ab}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{2-\delta_0}{\gamma_s k_c^2}\\
&\cdot\{\boldsymbol{M}_{\substack{e\\o}mn}(\gamma_f)[\alpha_{\substack{e\\o}mn}^{(fs)\mathrm{TE}}\boldsymbol{M}'_{\substack{e\\o}mn}(\gamma_s)\\
&+ (1-\delta_s^N)\beta_{\substack{e\\o}mn}^{(fs)\mathrm{TE}}\boldsymbol{M}'_{\substack{e\\o}mn}(-\gamma_s)]\\
&+ \boldsymbol{N}_{\substack{o\\e}mn}(\gamma_f)[\alpha_{\substack{e\\o}mn}^{(fs)\mathrm{TM}}\boldsymbol{N}'_{\substack{o\\e}mn}(\gamma_s)\\
&+ (1-\delta_s^N)\beta_{\substack{e\\o}mn}^{(fs)\mathrm{TM}}\boldsymbol{N}'_{\substack{o\\e}mn}(-\gamma_s)]\\
&+ (1-\delta_f^N)\boldsymbol{M}_{\substack{e\\o}mn}(-\gamma_f)[\alpha_{\substack{e\\o}mn}^{\prime(fs)\mathrm{TE}}\boldsymbol{M}'_{\substack{e\\o}mn}(\gamma_s)\\
&+ (1-\delta_s^N)\beta_{\substack{e\\o}mn}^{\prime(fs)\mathrm{TE}}\boldsymbol{M}'_{\substack{e\\o}mn}(-\gamma_s)]\\
&+ (1-\delta_f^N)\boldsymbol{N}_{\substack{o\\e}mn}(-\gamma_f)[\alpha_{\substack{e\\o}mn}^{\prime(fs)\mathrm{TM}}\boldsymbol{N}'_{\substack{o\\e}mn}(\gamma_s)\\
&+ (1-\delta_s^N)\beta_{\substack{e\\o}mn}^{\prime(fs)\mathrm{TM}}\boldsymbol{N}'_{\substack{o\\e}mn}(-\gamma_s)]\},\\
&\quad (z_0 < z < \infty; z_0 < z' < \infty)
\end{aligned} \tag{10}$$

where the symbol N appearing in the present and subsequent Kronecker deltas denotes the region number of the loaded waveguide. The coefficients $\alpha_{\substack{e\\o}mn}^{(fs)\mathrm{TE,TM}}$, $\beta_{\substack{e\\o}mn}^{(fs)\mathrm{TE,TM}}$, $\alpha_{\substack{e\\o}mn}^{\prime(fs)\mathrm{TE,TM}}$, and $\beta_{\substack{e\\o}mn}^{\prime(fs)\mathrm{TE,TM}}$ are to be determined from the boundary conditions.

IV. Coefficients of EM Types of Scattering Dyadic Green's Functions

Without any loss of generality, the electric and magnetic sources are assumed to be located in the sth region in the semi-infinite rectangular waveguide. Rewriting the boundary conditions satisfied by the electromagnetic types of dyadic Green's functions in (3a)–(3c) in matrix forms, we obtained the expressions. For the sake of succinct representation, we express the formulae for the coefficients of the electric (e) and magnetic (m) types of dyadic Green's functions as follows:

$$\begin{bmatrix} e^{i\gamma_1 z_0} & (\pm)(+-)e^{-i\gamma_1 z_0}\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}\alpha_{\substack{e\\o}mn}^{(1s)\mathrm{TE,TM}} & \beta_{\substack{e\\o}mn}^{(1s)\mathrm{TE,TM}}\\ \alpha_{\substack{e\\o}mn}^{\prime(1s)\mathrm{TE,TM}} + \delta_1^s & \beta_{\substack{e\\o}mn}^{\prime(1s)\mathrm{TE,TM}}\end{bmatrix} =, \tag{11a}$$

$$\begin{aligned}
\begin{bmatrix}\alpha_{\substack{e\\o}mn}^{[(\ell+1)s]\mathrm{TE,TM}} & \beta_{\substack{e\\o}mn}^{[(\ell+1)s]\mathrm{TE,TM}}\\ \alpha_{\substack{e\\o}mn}^{\prime[(\ell+1)s]\mathrm{TE,TM}} + \delta_{\ell+1}^s & \beta_{\substack{e\\o}mn}^{\prime[(\ell+1)s]\mathrm{TE,TM}}\end{bmatrix} &= \frac{1}{\mathcal{T}_{\substack{e\\o}mn}^{\ell\mathrm{TE,TM}}}\begin{bmatrix} e^{i(\gamma_\ell-\gamma_{\ell+1})z_\ell} & \mathcal{R}_{\substack{e\\o}mn}^{\ell\mathrm{TE,TM}}e^{-i(\gamma_\ell+\gamma_{\ell+1})z_\ell}\\ \mathcal{R}_{\substack{e\\o}mn}^{\ell\mathrm{TE,TM}}e^{i(\gamma_\ell+\gamma_{\ell+1})z_\ell} & e^{-i(\gamma_\ell-\gamma_{\ell+1})z_\ell}\end{bmatrix}\\
&\cdot\begin{bmatrix}\alpha_{\substack{e\\o}mn}^{(\ell s)\mathrm{TE,TM}} & \beta_{\substack{e\\o}mn}^{(\ell s)\mathrm{TE,TM}} + \delta_\ell^s\\ \alpha_{\substack{e\\o}mn}^{\prime(\ell s)\mathrm{TE,TM}} & \beta_{\substack{e\\o}mn}^{\prime(\ell s)\mathrm{TE,TM}}\end{bmatrix},
\end{aligned} \tag{11b}$$

$$\begin{aligned}
\begin{bmatrix}\alpha_{\substack{e\\o}mn}^{(Ns)\mathrm{TE,TM}} & \beta_{\substack{e\\o}mn}^{(Ns)\mathrm{TE,TM}}\\ \delta_N^s & 0\end{bmatrix} &= \frac{1}{\mathcal{T}_{\substack{e\\o}mn}^{(N-1)\mathrm{TE,TM}}}\begin{bmatrix} e^{i(\gamma_{N-1}-\gamma_N)z_{N-1}} & \mathcal{R}_{\substack{e\\o}mn}^{(N-1)\mathrm{TE,TM}}e^{-i(\gamma_{N-1}+\gamma_N)z_{N-1}}\\ \mathcal{R}_{\substack{e\\o}mn}^{(N-1)\mathrm{TE,TM}}e^{i(\gamma_{N-1}+\gamma_N)z_{N-1}} & e^{-i(\gamma_{N-1}-\gamma_N)z_{N-1}}\end{bmatrix}\\
&\cdot\begin{bmatrix}\alpha_{\substack{e\\o}mn}^{[(N-1)s]\mathrm{TE,TM}} & \beta_{\substack{e\\o}mn}^{[(N-1)s]\mathrm{TE,TM}} + \delta_{N-1}^s\\ \alpha_{\substack{e\\o}mn}^{\prime[(N-1)s]\mathrm{TE,TM}} & \beta_{\substack{e\\o}mn}^{\prime[(N-1)s]\mathrm{TE,TM}}\end{bmatrix}
\end{aligned} \tag{11c}$$

and (11b) and (11c), shown at the bottom of the previous page, where the notations $(\pm)$ and $(\mp)$ are designated for the subscript $\substack{e\\o}$ (corresponding to the *electric* and *magnetic* types of dyadic Green's functions, respectively), the notations $(+-)$ and $(-+)$ for the TE and TM waves, respectively, and $\mathcal{R}^{\ell\mathrm{TE,TM}}_{\substack{e\\o}mn}$ and $\mathcal{T}^{\ell\mathrm{TE,TM}}_{\substack{e\\o}mn}$ are given below in (12a)–(12d)

$$\mathcal{R}^{f\mathrm{TE}}_{\substack{e\\o}mn} = \frac{\wp_f\gamma_{f+1} - \wp_{f+1}\gamma_f}{\wp_f\gamma_{f+1} + \wp_{f+1}\gamma_f}, \tag{12a}$$

$$\mathcal{R}^{f\mathrm{TM}}_{\substack{e\\o}mn} = \frac{\wp_{f+1}\gamma_{f+1}k_f^2 - \wp_f\gamma_f k_{f+1}^2}{\wp_{f+1}\gamma_{f+1}k_f^2 + \wp_f\gamma_f k_{f+1}^2}, \tag{12b}$$

$$\mathcal{T}^{f\mathrm{TE}}_{\substack{e\\o}mn} = 1 + \mathcal{R}^{f\mathrm{TE}}_{\substack{e\\o}mn}, \tag{12c}$$

$$\mathcal{T}^{f\mathrm{TM}}_{\substack{e\\o}mn} = \frac{\wp_f k_{f+1}}{\wp_{f+1}k_f}\left(1 + \mathcal{R}^{f\mathrm{TM}}_{\substack{e\\o}mn}\right). \tag{12d}$$

The formulas of these coefficients can be derived generally from the above matrix equation system. For the sake of simplicity, the following inter-parameters are used in our representation [(13b) is shown at the bottom of the page]:

$$\boldsymbol{C}^{(\ell s)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \begin{bmatrix} \alpha^{(\ell s)\mathrm{TE,TM}}_{\substack{e\\o}mn} & \beta^{(\ell s)\mathrm{TE,TM}}_{\substack{e\\o}mn} \\ \alpha'^{(\ell s)\mathrm{TE,TM}}_{\substack{e\\o}mn} & \beta'^{(\ell s)\mathrm{TE,TM}}_{\substack{e\\o}mn} \end{bmatrix}, \tag{13a}$$

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}. \tag{13c}$$

For the sake of simplicity, we further define

$$\begin{aligned} \boldsymbol{F}^{(K)}_{\substack{e\\o}mn} &= \begin{bmatrix} (F^{(K)}_{\substack{e\\o}mn})_{11} & (F^{(K)}_{\substack{e\\o}mn})_{12} \\ (F^{(K)}_{\substack{e\\o}mn})_{21} & (F^{(K)}_{\substack{e\\o}mn})_{22} \end{bmatrix} \\ &= [\boldsymbol{T}^{(N-1)\mathrm{TE,TM}}_{\substack{e\\o}mn}][\boldsymbol{T}^{(N-2)\mathrm{TE,TM}}_{\substack{e\\o}mn}]\cdots[\boldsymbol{T}^{(K+1)\mathrm{TE,TM}}_{\substack{e\\o}mn}] \\ &\quad\cdot[\boldsymbol{T}^{(K)\mathrm{TE,TM}}_{\substack{e\\o}mn}]. \end{aligned} \tag{14}$$

Thus, the coefficients can be derived conveniently in the following three cases: $s = 1, s \neq 1, N$ and $s = N$.

A. The Source Located in the First Region

As the electric and magnetic current sources are located in the first layer, (11b) and (11c) can be written in the terse combined form

$$\begin{aligned} \boldsymbol{C}^{(N1)\mathrm{TE,TM}}_{\substack{e\\o}mn} &= \boldsymbol{T}^{(N-1)\mathrm{TE,TM}}_{\substack{e\\o}mn}\boldsymbol{T}^{(N-2)\mathrm{TE,TM}}_{\substack{e\\o}mn}\cdots\boldsymbol{T}^{(2)\mathrm{TE,TM}}_{\substack{e\\o}mn} \\ &\quad\cdot\boldsymbol{T}^{(1)\mathrm{TE,TM}}_{\substack{e\\o}mn}(\boldsymbol{C}^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} + \boldsymbol{A}_1) \\ &= \boldsymbol{F}^{(1)}_{\substack{e\\o}mn}(\boldsymbol{C}^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} + \boldsymbol{A}_1). \end{aligned} \tag{15}$$

Since two components of $\boldsymbol{C}^{(N1)\mathrm{TE,TM}}_{\substack{e\\o}mn}$ vanish, the coefficients for the first layer and the last layer can be obtained by solving (11a) and (15). After obtaining the coefficients for the first layer and the last layer, the results for the intermediate layers can be derived easily from the recurrent relation. The coefficients are given by

$$\alpha^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{(\mp)(+-)(F^{(1)}_{\substack{e\\o}mn})_{22}e^{-i\gamma_1 z_0}}{(F^{(1)}_{\substack{e\\o}mn})_{22}e^{i\gamma_1 z_0}(\mp)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}, \tag{16a}$$

$$\beta^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{(\pm)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}{(F^{(1)}_{\substack{e\\o}mn})_{22}e^{i\gamma_1 z_0}(\mp)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}, \tag{16b}$$

$$\alpha'^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{(\pm)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}{(F^{(1)}_{\substack{e\\o}mn})_{22}e^{i\gamma_1 z_0}(\mp)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}, \tag{16c}$$

$$\beta'^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{-(F^{(1)}_{\substack{e\\o}mn})_{21}e^{i\gamma_1 z_0}}{(F^{(1)}_{\substack{e\\o}mn})_{22}e^{i\gamma_1 z_0}(\mp)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}, \tag{16d}$$

$$\begin{aligned} \boldsymbol{C}^{(\ell 1)\mathrm{TE,TM}}_{\substack{e\\o}mn} &= \boldsymbol{T}^{(\ell-1)\mathrm{TE,TM}}_{\substack{e\\o}mn}\boldsymbol{T}^{(\ell-2)\mathrm{TE,TM}}_{\substack{e\\o}mn}\cdots\boldsymbol{T}^{(2)\mathrm{TE,TM}}_{\substack{e\\o}mn} \\ &\quad\cdot\boldsymbol{T}^{(1)\mathrm{TE,TM}}_{\substack{e\\o}mn}(\boldsymbol{C}^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} + \boldsymbol{A}_1), \end{aligned} \tag{17}$$

and

$$\begin{aligned} \alpha^{(N1)\mathrm{TE,TM}}_{\substack{e\\o}mn} &= (F^{(1)}_{\substack{e\\o}mn})_{11}\alpha^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} + (F^{(1)}_{\substack{e\\o}mn})_{12} \\ &\quad\cdot\alpha'^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn}, \end{aligned} \tag{18a}$$

$$\begin{aligned} \beta^{(N1)\mathrm{TE,TM}}_{\substack{e\\o}mn} &= (F^{(1)}_{\substack{e\\o}mn})_{11}\beta^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} + (F^{(1)}_{\substack{e\\o}mn})_{12}\beta'^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} \\ &\quad + (F^{(1)}_{\substack{e\\o}mn})_{11}. \end{aligned} \tag{18b}$$

B. The Source Located in the Intermediate Region

To derive the coefficients in each layer is not so easy when the sources are located in the intermediate layers. From (11b) and (11c), we have

$$\begin{aligned} \boldsymbol{C}^{(Ns)\mathrm{TE,TM}}_{\substack{e\\o}mn} &= \boldsymbol{T}^{(N-1)\mathrm{TE,TM}}_{\substack{e\\o}mn}\boldsymbol{T}^{(N-2)\mathrm{TE,TM}}_{\substack{e\\o}mn}\cdots\boldsymbol{T}^{(s)\mathrm{TE,TM}}_{\substack{e\\o}mn} \\ &\quad\cdot[\boldsymbol{T}^{(s-1)\mathrm{TE,TM}}_{\substack{e\\o}mn}\cdots\boldsymbol{T}^{(2)\mathrm{TE,TM}}_{\substack{e\\o}mn}\boldsymbol{T}^{(1)\mathrm{TE,TM}}_{\substack{e\\o}mn} \\ &\quad\cdot\boldsymbol{C}^{(1s)\mathrm{TE,TM}}_{\substack{e\\o}mn} + (\boldsymbol{A}_1 - \boldsymbol{A}_2)] \\ &= \boldsymbol{F}^{(1)}_{\substack{e\\o}mn}\boldsymbol{C}^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} + \boldsymbol{F}^{(s)}_{\substack{e\\o}mn}(\boldsymbol{A}_1 - \boldsymbol{A}_2). \end{aligned} \tag{19}$$

Furthermore, the whole set of coefficients of the scattering dyadic Green's functions can be given when the sources are located in the sth intermediate layer by

$$\alpha^{(1s)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{(\mp)(+-)(F^{(s)}_{\substack{e\\o}mn})_{22}e^{-i\gamma_1 z_0}}{(F^{(1)}_{\substack{e\\o}mn})_{22}e^{i\gamma_1 z_0}(\mp)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}, \tag{20a}$$

$$\beta^{(1s)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{(\pm)(+-)(F^{(s)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}{(F^{(1)}_{\substack{e\\o}mn})_{22}e^{i\gamma_1 z_0}(\mp)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}, \tag{20b}$$

$$\boldsymbol{T}^{(\ell)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{1}{\mathcal{T}^{\ell\mathrm{TE,TM}}_{\substack{e\\o}mn}}\begin{bmatrix} e^{i(\gamma_\ell-\gamma_{\ell+1})z_\ell} & \mathcal{R}^{\ell\mathrm{TE,TM}}_{\substack{e\\o}mn}e^{-i(\gamma_\ell+\gamma_{\ell+1})z_\ell} \\ \mathcal{R}^{\ell\mathrm{TE,TM}}_{\substack{e\\o}mn}e^{i(\gamma_\ell+\gamma_{\ell+1})z_\ell} & e^{-i(\gamma_\ell-\gamma_{\ell+1})z_\ell} \end{bmatrix} \tag{13b}$$

$$\alpha'^{(1s)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{(F^{(s)}_{\substack{e\\o}mn})_{22}e^{i\gamma_1 z_0}}{(F^{(1)}_{\substack{e\\o}mn})_{22}e^{i\gamma_1 z_0}(\mp)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}, \tag{20c}$$

$$\beta'^{(1s)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{-(F^{(s)}_{\substack{e\\o}mn})_{21}e^{i\gamma_1 z_0}}{(F^{(1)}_{\substack{e\\o}mn})_{22}e^{i\gamma_1 z_0}(\mp)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}, \tag{20d}$$

$$\begin{aligned} \boldsymbol{C}^{(\ell s)\mathrm{TE,TM}}_{\substack{e\\o}mn} =& \boldsymbol{T}^{(\ell-1)\mathrm{TE,TM}}_{\substack{e\\o}mn}\cdots \\ &\cdot \boldsymbol{T}^{(s)\mathrm{TE,TM}}_{\substack{e\\o}mn}(\boldsymbol{T}^{(s-1)\mathrm{TE,TM}}_{\substack{e\\o}mn}\cdots \boldsymbol{T}^{(1)\mathrm{TE,TM}}_{\substack{e\\o}mn} \\ &\cdot \boldsymbol{C}^{(1s)\mathrm{TE,TM}}_{\substack{e\\o}mn} + H(\ell-s-1)\boldsymbol{A}_1 \\ &- H(\ell-s)\boldsymbol{A}_2) \end{aligned} \tag{21}$$

and

$$\begin{aligned} \alpha^{(Ns)\mathrm{TE,TM}}_{\substack{e\\o}mn} =& (F^{(1)}_{\substack{e\\o}mn})_{11}\alpha^{(1s)\mathrm{TE,TM}}_{\substack{e\\o}mn} + (F^{(1)}_{\substack{e\\o}mn})_{12} \\ &\cdot \alpha'^{(1s)\mathrm{TE,TM}}_{\substack{e\\o}mn} - (F^{(s)}_{\substack{e\\o}mn})_{12}, \end{aligned} \tag{22a}$$

$$\begin{aligned} \beta^{(Ns)\mathrm{TE,TM}}_{\substack{e\\o}mn} =& (F^{(1)}_{\substack{e\\o}mn})_{11}\beta^{(1s)\mathrm{TE,TM}}_{\substack{e\\o}mn} + (F^{(1)}_{\substack{e\\o}mn})_{12} \\ &\cdot \beta'^{(1s)\mathrm{TE,TM}}_{\substack{e\\o}mn} + (F^{(s)}_{\substack{e\\o}mn})_{11} \end{aligned} \tag{22b}$$

where $H(\ell - \ell_0)$ (equal to 1 for $\ell \geq \ell_0$ and 0 for $\ell < \ell_0$) denotes the step function.

C. The Source Located in the Last Region

Following the procedure similar to the above, we have

$$\begin{aligned} \boldsymbol{C}^{(NN)\mathrm{TE,TM}}_{\substack{e\\o}mn} =& \boldsymbol{T}^{(N-1)\mathrm{TE,TM}}_{\substack{e\\o}mn}\boldsymbol{T}^{(N-2)\mathrm{TE,TM}}_{\substack{e\\o}mn}\cdots \boldsymbol{T}^{(2)\mathrm{TE,TM}}_{\substack{e\\o}mn} \\ &\cdot \boldsymbol{T}^{(1)\mathrm{TE,TM}}_{\substack{e\\o}mn}\boldsymbol{C}^{(1N)\mathrm{TE,TM}}_{\substack{e\\o}mn} - \boldsymbol{A}_2 \\ =& \boldsymbol{F}^{(1)}_{\substack{e\\o}mn}\boldsymbol{C}^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} - \boldsymbol{A}_2. \end{aligned} \tag{23a}$$

When the EM sources are located in the last layer, only half of the coefficients for the scattering dyadic Green's functions need to be obtained. Therefore, the coefficients can be given as follows:

$$\alpha^{(1N)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{(\mp)(+-)e^{-i\gamma_1 z_0}}{(F^{(1)}_{\substack{e\\o}mn})_{22}e^{i\gamma_1 z_0}(\mp)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}, \tag{24a}$$

$$\alpha'^{(1N)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{e^{i\gamma_1 z_0}}{(F^{(1)}_{\substack{e\\o}mn})_{22}e^{i\gamma_1 z_0}(\mp)(+-)(F^{(1)}_{\substack{e\\o}mn})_{21}e^{-i\gamma_1 z_0}}, \tag{24b}$$

$$\begin{aligned} \boldsymbol{C}^{(\ell s)\mathrm{TE,TM}}_{\substack{e\\o}mn} =& \boldsymbol{T}^{(\ell-1)\mathrm{TE,TM}}_{\substack{e\\o}mn}\cdots \boldsymbol{T}^{(s)\mathrm{TE,TM}}_{\substack{e\\o}mn}(\boldsymbol{T}^{(s-1)\mathrm{TE,TM}}_{\substack{e\\o}mn}\cdots \\ &\cdot \boldsymbol{T}^{(1)\mathrm{TE,TM}}_{\substack{e\\o}mn}\boldsymbol{C}^{(1N)\mathrm{TE,TM}}_{\substack{e\\o}mn}) \end{aligned} \tag{25}$$

and

$$\begin{aligned} \alpha^{(NN)\mathrm{TE,TM}}_{\substack{e\\o}mn} =& (F^{(1)}_{\substack{e\\o}mn})_{11}\alpha^{(1N)\mathrm{TE,TM}}_{\substack{e\\o}mn} + (F^{(1)}_{\substack{e\\o}mn})_{12} \\ &\cdot \alpha'^{(1N)\mathrm{TE,TM}}_{\substack{e\\o}mn}. \end{aligned} \tag{26}$$

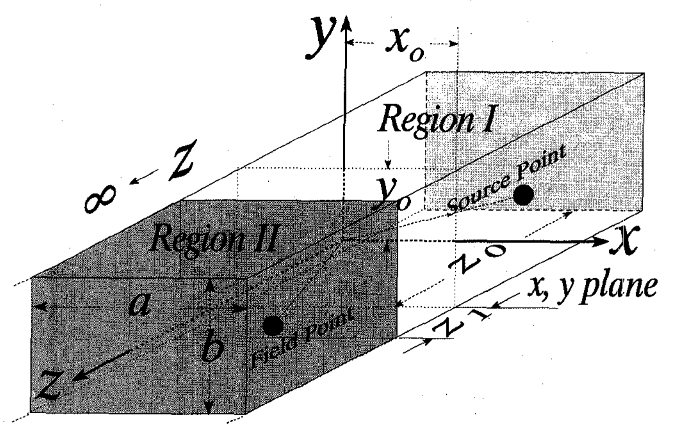

Fig. 2. Geometry of the semi-infinite rectangular waveguide with one load.

V. Application to Semi-Infinite Waveguide with One Load

The dyadic Green's functions have different mathematical forms when the current source is located in the different regions of the waveguide. When the semi-infinite waveguide has one load, the waveguide can be considered as one filled with a two-layered medium. In this case, the dyadic Green's functions can be reduced from (15) in sequence in the order of the source present in regions I and II

$$\begin{aligned} \boldsymbol{F}^{(1)}_{\substack{e\\o}mn} =& \boldsymbol{T}^{(1)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{1}{\mathcal{T}^{1\mathrm{TE,TM}}_{\substack{e\\o}mn}} \\ &\cdot \begin{bmatrix} e^{i(\gamma_1-\gamma_2)z_1} & \mathcal{R}^{1\mathrm{TE,TM}}_{\substack{e\\o}mn}e^{-(i\gamma_1+\gamma_2)z_1} \\ \mathcal{R}^{1\mathrm{TE,TM}}_{\substack{e\\o}mn}e^{i(\gamma_1+\gamma_2)z_1} & e^{-i(\gamma_1-\gamma_2)z_1} \end{bmatrix}, \end{aligned} \tag{27a}$$

$$\boldsymbol{F}^{(2)}_{\substack{e\\o}mn} = \boldsymbol{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{27b}$$

A. Excitation Source Present in Region I

The scattering dyadic Green's function represents the contribution due to the presence of the waveguide interfaces perpendicular to z-direction. Thus, taking the reflected waves into account when the source is located in region I shown in Fig. 2 (i.e., $s = 1$), we may construct the scattering Green dyadic as follows according to (10):

for $f = 1$,

$$\begin{aligned} \overline{\boldsymbol{G}}^{(11)}_{\substack{e\\m}S}(\boldsymbol{r},\boldsymbol{r}') =& \frac{i}{ab}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{2-\delta_0}{\gamma_1 k_c^2} \\ &\cdot \{\boldsymbol{M}_{\substack{e\\o}mn}(\gamma_1)[\alpha^{(11)\mathrm{TE}}_{\substack{e\\o}mn}\boldsymbol{M}'_{\substack{e\\o}mn}(\gamma_1) \\ &+ \beta^{(11)\mathrm{TE}}_{\substack{e\\o}mn}\boldsymbol{M}'_{\substack{e\\o}mn}(-\gamma_1)] \\ &+ \boldsymbol{N}_{\substack{o\\e}mn}(\gamma_1)[\alpha^{(11)\mathrm{TM}}_{\substack{e\\o}mn}\boldsymbol{N}'_{\substack{o\\e}mn}(\gamma_1) \\ &+ \beta^{(11)TM}_{\substack{e\\o}mn}\boldsymbol{N}'_{\substack{o\\e}mn}(-\gamma_1)] \\ &+ \boldsymbol{M}_{\substack{e\\o}mn}(-\gamma_1)[\alpha'^{(11)\mathrm{TE}}_{\substack{e\\o}mn}\boldsymbol{M}'_{\substack{e\\o}mn}(\gamma_1) \\ &+ \beta'^{(11)\mathrm{TE}}_{\substack{e\\o}mn}\boldsymbol{M}'_{\substack{e\\o}mn}(-\gamma_1)] \\ &+ \boldsymbol{N}_{\substack{o\\e}mn}(-\gamma_1)[\alpha'^{(11)\mathrm{TM}}_{\substack{e\\o}mn}\boldsymbol{N}'_{\substack{o\\e}mn}(\gamma_1) \\ &+ \beta'^{(11)\mathrm{TM}}_{\substack{e\\o}mn}\boldsymbol{N}'_{\substack{o\\e}mn}(-\gamma_1)]\}, \\ &(z_0 < z < z_1; z_0 < z' < z_1) \end{aligned} \tag{28a}$$

and for $f = 2$,

$$\overline{G}^{(21)}_{\substack{e\\m}S}(\boldsymbol{r},\boldsymbol{r}') = \frac{i}{ab}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{2-\delta_0}{\gamma_1 k_c^2} \cdot \{\boldsymbol{M}_{\substack{e\\o}mn}(\gamma_2)[\alpha^{(21)\mathrm{TE}}_{\substack{e\\o}mn}\boldsymbol{M}'_{\substack{e\\o}mn}(\gamma_1) + \beta^{(21)\mathrm{TE}}_{\substack{e\\o}mn}\boldsymbol{M}'_{\substack{e\\o}mn}(-\gamma_1)] + \boldsymbol{N}_{\substack{o\\e}mn}(\gamma_2)[\alpha^{(21)\mathrm{TM}}_{\substack{e\\o}mn}\boldsymbol{N}'_{\substack{o\\e}mn}(\gamma_1) + \beta^{(21)\mathrm{TM}}_{\substack{e\\o}mn}\boldsymbol{N}'_{\substack{o\\e}mn}(-\gamma_1)]\}, \quad (z_1 < z < \infty; z_0 < z' < z_1) \tag{28b}$$

where the coefficients $\alpha^{(fs)\mathrm{TE,TM}}_{\substack{e\\o}mn}$, $\beta^{(fs)\mathrm{TE,TM}}_{\substack{e\\o}mn}$, $\alpha'^{(fs)\mathrm{TE,TM}}_{\substack{e\\o}mn}$, and $\beta^{(fs)\mathrm{TE,TM}}_{\substack{e\\o}mn}$ are determined from the boundary conditions.

From the boundary conditions in (3a)–(3c), the general solutions of these coefficients of scattering dyadic Green's functions in each of the regions have been obtained in (16a)–(18b) when the EM current sources are located in region I. The unknowns in (28) are coefficients for the special case and can be reduced directly from the general solutions obtained in (16a)–(18b). When the excitation source is located in the first region, six sets of coefficients of the scattering Green dyadic are derived. They are given, for region I, by

$$\alpha^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} = (\mp)(+-)\frac{1}{\mathcal{D}^{\mathrm{TE,TM}}_{\substack{e\\o}mn}}e^{-i2\gamma_1 z_0}, \tag{29a}$$

$$\beta^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} = (\pm)(+-)\frac{\mathcal{R}^{1\mathrm{TE,TM}}_{\substack{e\\o}mn}}{\mathcal{D}^{\mathrm{TE,TM}}_{\substack{e\\o}mn}}e^{i2\gamma_1(z_1-z_0)}, \tag{29b}$$

$$\alpha'^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} = (\pm)(+-)\frac{\mathcal{R}^{1\mathrm{TE,TM}}_{\substack{e\\o}mn}}{\mathcal{D}^{\mathrm{TE,TM}}_{\substack{e\\o}mn}}e^{i2\gamma_1(z_1-z_0)}, \tag{29c}$$

$$\beta'^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn} = -\frac{\mathcal{R}^{1\mathrm{TE,TM}}_{\substack{e\\o}mn}}{\mathcal{D}^{\mathrm{TE,TM}}_{\substack{e\\o}mn}}e^{i2\gamma_1 z_1} \tag{29d}$$

and for region II

$$\alpha^{(21)\mathrm{TE,TM}}_{\substack{e\\o}mn} = (\mp)(+-)\frac{\mathcal{T}^{*1\mathrm{TE,TM}}_{\substack{e\\o}mn}}{\mathcal{D}^{\mathrm{TE,TM}}_{\substack{e\\o}mn}}e^{i(\gamma_1-\gamma_2)z_1 - i2\gamma_1 z_0}, \tag{30a}$$

$$\beta^{(21)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{\mathcal{T}^{*1\mathrm{TE,TM}}_{\substack{e\\o}mn}}{\mathcal{D}^{\mathrm{TE,TM}}_{\substack{e\\o}mn}}e^{i(\gamma_1-\gamma_2)z_1} \tag{30b}$$

where

$$\mathcal{R}^{1,2\mathrm{TE}}_{\substack{e\\o}mn} = \frac{\wp_{1,2}\gamma_{2,3} - \wp_{2,3}\gamma_{1,2}}{\wp_{1,2}\gamma_{2,3} + \wp_{2,3}\gamma_{1,2}}, \tag{31a}$$

$$\mathcal{R}^{1,2\mathrm{TM}}_{\substack{e\\o}mn} = \frac{\wp_{2,3}\gamma_{2,3}k_{1,2}^2 - \wp_{1,2}\gamma_{1,2}k_{2,3}^2}{\wp_{2,3}\gamma_{2,3}k_{1,2}^2 + \wp_{1,2}\gamma_{1,2}k_{2,3}^2}, \tag{31b}$$

$$\mathcal{T}^{*1,2\mathrm{TE}}_{\substack{e\\o}mn} = 1 - \mathcal{R}^{1,2\mathrm{TE}}_{\substack{e\\o}mn}, \tag{31c}$$

$$\mathcal{T}^{*1,2\mathrm{TM}}_{\substack{e\\o}mn} = \frac{k_{1,2}\wp_{2,3}}{k_{2,3}\wp_{1,2}}(1 - \mathcal{R}^{1,2\mathrm{TM}}_{\substack{e\\o}mn}), \tag{31d}$$

$$\mathcal{D}^{\mathrm{TE,TM}}_{\substack{e\\o}mn} = 1(\mp)(+-)\mathcal{R}^{1\mathrm{TE,TM}}_{\substack{e\\o}mn}e^{i2\gamma_1(z_1-z_0)}. \tag{31e}$$

It should be pointed out that the parameters $\mathcal{T}^{*(1,2)(\mathrm{TE,TM})}_{\substack{e\\o}mn}$ are not the same as $\mathcal{T}^{(1,2)(\mathrm{TE,TM})}_{\substack{e\\o}mn}$ in (12c) and (12d). It should also be pointed out that the same expression of the coefficients $\beta^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn}$ and $\alpha'^{(11)\mathrm{TE,TM}}_{\substack{e\\o}mn}$ shows the symmetry of the dyadic Green's functions.

B. Excitation Source Present in Region II

Using a method similar to the above, the scattering dyadic Green's function in the case of the sources located in region II shown in Fig. 2 (i.e., $s = 2$) can be written as follows for: $f = 1$,

$$\overline{G}^{(12)}_{\substack{e\\m}S}(\boldsymbol{r},\boldsymbol{r}') = \frac{i}{ab}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{2-\delta_0}{\gamma_2 k_c^2} \cdot \{[\alpha^{(12)\mathrm{TE}}_{\substack{e\\o}mn}\boldsymbol{M}_{\substack{e\\o}mn}(\gamma_1) + \alpha'^{(12)\mathrm{TE}}_{\substack{e\\o}mn} \cdot \boldsymbol{M}_{\substack{e\\o}mn}(-\gamma_1)]\boldsymbol{M}'_{\substack{e\\o}mn}(\gamma_2) + [\alpha^{(12)\mathrm{TM}}_{\substack{e\\o}mn}\boldsymbol{N}_{\substack{o\\e}mn}(\gamma_1) + \alpha'^{(12)\mathrm{TM}}_{\substack{e\\o}mn} \cdot \boldsymbol{N}_{\substack{o\\e}mn}(-\gamma_1)]\boldsymbol{N}'_{\substack{o\\e}mn}(\gamma_2)\}, \quad (z_0 < z < z_1; z_1 < z' < \infty) \tag{32a}$$

and for $f = 2$

$$\overline{G}^{(22)}_{\substack{e\\m}S}(\boldsymbol{r},\boldsymbol{r}') = \frac{i}{ab}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{2-\delta_0}{\gamma_2 k_c^2} \cdot [\alpha^{(22)\mathrm{TE}}_{\substack{e\\o}mn}\boldsymbol{M}_{\substack{e\\o}mn}(\gamma_2)\boldsymbol{M}'_{\substack{e\\o}mn}(\gamma_2) + \alpha^{(22)\mathrm{TM}}_{\substack{e\\o}mn} \cdot \boldsymbol{N}_{\substack{o\\e}mn}(\gamma_2)\boldsymbol{N}'_{\substack{o\\e}mn}(\gamma_2)], \quad (z_1 < z < \infty; z_1 < z' < \infty) \tag{32b}$$

where the coefficients $\alpha^{(fs)\mathrm{TE,TM}}_{\substack{e\\o}mn}$, $\beta^{(fs)\mathrm{TE,TM}}_{\substack{e\\o}mn}$, $\alpha'^{(fs)\mathrm{TE,TM}}_{\substack{e\\o}mn}$, and $\beta^{(fs)\mathrm{TE,TM}}_{\substack{e\\o}mn}$ are determined from the boundary conditions. These three sets of coefficients are given, for region I, by

$$\alpha^{(12)\mathrm{TE,TM}}_{\substack{e\\o}mn} = (\mp)(+-)\frac{\mathcal{T}^{1\mathrm{TE,TM}}_{\substack{e\\o}mn}}{\mathcal{D}^{\mathrm{TE,TM}}_{\substack{e\\o}mn}}e^{i(\gamma_1-\gamma_2)z_1 - 2\gamma_1 z_0}, \tag{33a}$$

$$\alpha'^{(12)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{\mathcal{T}^{1\mathrm{TE,TM}}_{\substack{e\\o}mn}}{\mathcal{D}^{\mathrm{TE,TM}}_{\substack{e\\o}mn}}e^{i(\gamma_1-\gamma_2)z_1}, \tag{33b}$$

and for region II

$$\alpha^{(22)\mathrm{TE,TM}}_{\substack{e\\o}mn} = \frac{\mathcal{R}^{1\mathrm{TE,TM}}_{\substack{e\\o}mn}(\mp)(+-)e^{i2\gamma_1(z_1-z_0)}}{\mathcal{D}^{\mathrm{TE,TM}}_{\substack{e\\o}mn}}e^{-i2\gamma_2 z_1}. \tag{34}$$

VI. Discussion and Conclusion

A. Comparison with Jarem's Results

Previously, Jarem [7] derived the $\hat{\boldsymbol{y}}\hat{\boldsymbol{y}}$-component of the dyadic Green's function for defining the electromagnetic fields due to an electric current distribution in a semi-infinite rectangular waveguide with one load. Since the probe used in [7] is located in region I, the dyadic Green's function that Jarem derived should be the one associated with (28a). Although Jarem also used the principle of scattering superposition to construct the needed dyadic Green's function, his idea to achieve the goal is slightly different from the one presented here. The difference can be seen from the following aspects.

The Green dyadic in the semi-infinite rectangular waveguide with loads usually consists of two parts. Jarem assumed that the principal part corresponding to $\Psi_S(z,z')$ is the Green dyadic in the semi-infinite rectangular waveguide without load and the additional part corresponding to $\Psi_L^{\rm TE}(z,z')$ (for TE waves) and $\Psi_L^{\rm TM}(z,z')$ (for TM waves) results from the transmission and reflection due to the semi-infinite load's interface. This paper assumes that the principal part is the Green dyadic in the rectangular infinite waveguide without load and the additional part results from the transmission and reflection due to all the interfaces perpendicular to z-direction.

From (9) and (28a), the $\hat{y}\hat{y}$-component of the Green dyadic can be written as follows by letting $k_1 = k_0$, and $\gamma_{nm} = i\gamma_1 = \sqrt{k_c^2 - k_1^2}$ and taking only the electric type of the Green dyad

$$\begin{aligned} G_{yy} = &-\frac{2}{ab}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{(2-\delta_m^0)}{\gamma_{nm}k_c^2}(\Psi_S(z,z') + \Psi_L^{\rm TE}(z,z') \\ &+ \Psi_L^{\rm TM}(z,z'))\sin\frac{n\pi(x+x_0)}{a}\sin\frac{n\pi(x'+x_0)}{a} \\ &\cdot\cos\frac{m\pi(y+y_0)}{b}\cos\frac{m\pi(y'+y_0)}{b} \end{aligned} \tag{35}$$

where

$$\begin{aligned} \Psi_S(z,z') = &\frac{[(m\pi/b)^2 - k_1^2]k_c^2}{k_1^2} \\ &\cdot\begin{cases} e^{\gamma_{nm}(z-z_0)}\sinh[\gamma_{nm}(z'-z_0)] & z \ge z' \\ \sinh[\gamma_{nm}(z-z_0)]e^{\gamma_{nm}(z'-z_0)} & z < z', \end{cases} \end{aligned} \tag{36a}$$

$$\begin{aligned} \Psi_L^{\rm TE}(z,z') = &\left(\frac{n\pi}{a}\right)^2\frac{2\Gamma^{\rm TE}e^{2\gamma_{nm}(z_1-z_0)}}{1+\Gamma^{\rm TE}e^{2\gamma_{nm}(z_1-z_0)}} \\ &\cdot\sinh[\gamma_{nm}(z-z_0)]\sinh[\gamma_{nm}(z'-z_0)], \end{aligned} \tag{36b}$$

$$\begin{aligned} \Psi_L^{\rm TM}(z,z') = &\left(\frac{m\pi}{b}\right)^2\left(\frac{\gamma_1}{k_1}\right)^2\frac{2\Gamma^{\rm TM}e^{2\gamma_{nm}(z_1-z_0)}}{1+\Gamma^{\rm TM}e^{2\gamma_{nm}(z_1-z_0)}} \\ &\cdot\sinh[\gamma_{nm}(z-z_0)]\sinh[\gamma_{nm}(z'-z_0)], \end{aligned} \tag{36c}$$

$$\Gamma^{\rm TE} = -\frac{\mu_1\gamma_2 - \mu_2\gamma_1}{\mu_1\gamma_2 + \mu_2\gamma_1}, \tag{36d}$$

$$\Gamma^{\rm TM} = \frac{\mu_2\gamma_2 k_1^2 - \mu_1\gamma_1 k_2^2}{\mu_2\gamma_2 k_1^2 + \mu_1\gamma_1 k_2^2}. \tag{36e}$$

The result presented by (35) together with (36a)–(36e) is more complete and generalized than the Green function given by Jarem [7] because the result obtained here is a full-wave analytical solution of G_{yy} and no approximation has been made in the above expression. The expression G_{yy}^A given by Jarem [7] does not have the contribution from the TM modes and higher-order TE modes as well. If we assume that only TE_{10}-mode waves, i.e $(n,m) = (1,0)$ in (35) together with (36a)–(36e), propagate in the semi-infinite waveguide with a load, the approximate results are exactly Jarem's formula of G_{yy}^A [7], except that the reflection coefficients for the TE modes of different orders are assumed to be the same. Certainly, both the reflection and transmission coefficients of the TE and TM modes should be different, as can be seen from the results presented here and those given by Tai [1] for a medium of various geometries. This shows the applicability and generalization of the method presented.

B. Reduction to Semi-infinite Waveguide without Load

The dyadic Green's functions are obtained for defining the electromagnetic fields due to the electric and magnetic current sources located in the semi- infinite rectangular waveguide with multiple loads. If the loads disappear on the assumption that the multi-layered media have the same dielectric constant, the derived general formulae of dyadic Green's functions for the waveguide with multiple loads should reduce directly to the results for the waveguide without load. To demonstrate how the coefficients of the dyadic Green's functions are reduced to the simple forms, we use the expression obtained in (29a)–(30b) and (33a)–(34) as an example.

In fact, $\gamma_1 = \gamma_2$ when the different layers have the same dielectric constant. Thus, it can be seen from (31a)–(31e) that

$$\mathcal{R}_{\substack{e\\o}mn}^{1{\rm TE,TM}} = 0, \quad \mathcal{T}_{\substack{e\\o}mn}^{*1{\rm TE,TM}} = 1, \quad \mathcal{D}_{\substack{e\\o}mn}^{\rm TE,TM} = 1.$$

Furthermore, from (10), we can see that in the expression of the dyadic Green's function in (32b), only the terms containing the coefficients $\alpha_{\substack{e\\o}mn}^{\rm TE,TM}$ exist. From (29a), (30a), (33a) or (34), we find

$$\alpha_{\substack{e\\o}mn}^{\rm TE,TM} = (\mp)(+-)e^{-i2\gamma z_0}. \tag{37}$$

This result agrees very well with the electric and magnetic types of Green dyads presented earlier in [5], which once again demonstrates the applicability of the present method.

C. Reduction to Rectangular Cavity Case

Furthermore, we may reduce the formulae of dyadic Green's functions and their coefficients for the semi-infinite rectangular waveguide to those for the rectangular cavity. Region I of the geometry shown in Fig. 2 can be considered as a rectangular cavity when the load of the semi-infinite rectangular waveguide is filled with conducting material. This reduction can be achieved easily by assuming that $\sigma \to \infty$. That means

$$\varepsilon_2 = \varepsilon\left(1 + \frac{i\sigma}{\omega\varepsilon}\right) \simeq \frac{i\sigma}{\omega\varepsilon} \overset{\sigma\to\infty}{\longrightarrow} i\infty.$$

Thus, the coefficients for such a cavity can be obtained from (29a)–(29d). It has been proven that the coefficients reduced here are exactly the same as those obtained by Li *et al.* [5] from boundary conditions.

From the above discussions, we may draw the following conclusions. The electric and magnetic types of dyadic Green's functions for a rectangular waveguide with multiple loads have been analytically formulated from the corresponding boundary conditions for the general case. Rigorous and general expressions of the dyadic Green's functions and their coefficients are obtained in terms of recurrent transmission matrices. By reducing these general expressions, the dyadic Green's functions and their coefficients for the semi-infinite rectangular waveguide with one load are further derived and compared with the $\hat{y}\hat{y}$-component presented in previously published work. The results presented here are more complete than Jarem's results [7]. After reduction of the general results for the special cases, the good agreement 1) between Jarem's $\hat{y}\hat{y}$-component of dyadic Green's functions and the results specially reduced here, and 2) between the dyadic Green's functions presented earlier by Li *et al.* [5] and the dyadic Green's function reduced here from the general formulae, has demonstrated the applicability of both the results obtained and the method presented. A full-wave numerical analysis of input impedance of a probe-excited semi-infinite rectangular waveguide will be carried out in Part II of this paper by using the dyadic Green's functions presented in this part.

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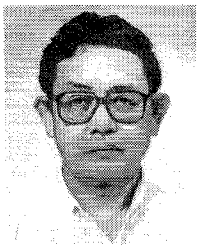

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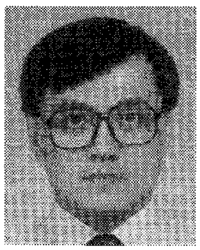

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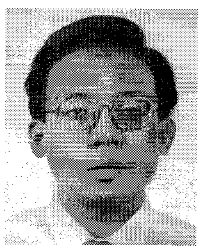

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